Number and Measure in Economics

A Critical Examination of Mathematical Method and of Mathematical Price Theory*

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I.

The example of the natural sciences, which owe their success largely to the use of numbers and measures, has also spurred the other sciences to attempt to use the same tools. The epithet "exact" has been reserved for those sciences that have advanced to the point of numerical precision and thus the use of number made a required precondition for attainment of the highest grade of scientific knowledge. Whether legitimately may remain undecided; in any case, the various efforts to meet the precondition demonstrate that its legitimacy is accepted by many. Even the human sciences are not by this current left untouched. Psychophysics attests to this.

But there is also no shortage of protests against this line of thought. One is rightly not satisfied with the reference to the example of the natural sciences, but requires proof in every single case that the use of numbers and measures is not only possible but also beneficial; because every science has its own peculiarity.

Economics in particular has a peculiar position within the sciences with regard to the use of measure and number. On the one hand, through extensive use of statistics, it too has given number a great deal of latitude and has undoubtedly gained precision and applicability; on the other hand, the very line of thought that aspired to a living grasp of full reality is quite opposed to the use of numbers in theoretical economics and mathematical deduction. It has contrasted the mathematical method as the extremely deductive with the historical method. In order to resolve this apparent contradiction, according to which number on the one hand brings reality nearer, on the other hand seems to be removed from it, it is necessary to grasp the relationship of scientific knowledge to reality more exactly.

Every science has the aim to become applied, however far it may remain from the practical purposes of daily life. Their general propositions only have sense and significance because they can be filled with concrete content from reality. In this sense, every science is an empirical science, even mathematics,

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whose formulas and sentences are intended to be applied to quantities obtained by measuring real objects. Economics with its complex dimensions requires its own science of measuring these, statistics. This provides the data that are necessary for the application of economics to reality. For its part it demands — and probably also contributes to the fulfillment of this requirement — that the terms intended for application to reality are capable of numerical determination, that they are quantitatively defined. Terms such as "population", "mortality", "wealth distribution", "large business", and so forth it demands be translated from the vague language of everyday life into the strict language of science. The necessity and possibility of such an application of number and measure cannot be doubted. Calculations of larger and smaller amounts can also follow the statistics, thus creating an area of application of mathematics to economic objects. But this application is not intended when mathematical method in economics is disputed. One has then not an operation with numbers at the conclusion of the consideration, at the transition in sense from theory to reality, but employment of mathematical deduction fron the outset, from the principles. The extreme representatives of the mathematical approach say that economics is a mathematical science, that is a science whose consclusions can be deduced from principles through purely mathematical means.

It would be a gratuitous undertaking to refute this claim; for anyone who is only remotely familiar the problems that the science of today faces will not arrive at the thought to intend to solve them by mathematical deduction. If we nonetheless examine more closely the applicability of the mathematical method, it happens thus because we consider the question of rejecting the most extreme claim not yet settled. There remain more modest claims, the correctness of which should be examined; even if we were obliged to deny legitimacy to any mathematical deduction, there is always still the question to decide of whether the basic economic concepts are at all measurable quantities; and finally we gladly use this opportunity to put the general question of method from this perspective into what seems to us a new light.

In regard to the data for application to reality, as we have seen, every science is entirely dependent on experience. It can in this regard never even only come close to a final conclusion, but it is always obliged to employ observations and measurements. However there are sciences that in regard to their principles are capable of assured propositions. It is the deductive sciences in the narrow sense, mathematics at the top. Into the philosophic controversy over whether these principles, like data, need to be inferred from experience, we need not here enter; it suffices that the purely deductive sciences are able to enumerate their principles completely (if not perfectly) and from them — apart from technical difficulties — can derive all propositions that could possibly come in the applications in questions of reality. All remaining sciences, however, may they themselves employ the deductive method in still so great extent, can never arrive at a complete enumerations of their principles sufficient for all applications. From time to time new cases occur in them, new phenomena, which for their explanation require the introduction of new principles, and the series of these may thus no more ever become regarded as self-contained than one may

assume that one can ever come to the end of new applications. The differences of the sciences highlighted here express themselves also in the boundaries of their governed domains. The purely deductive are sufficiently defined just by the principles and their areas of application, while the remaining sciences must be defined at least also factually, that is by specification of the objects with which they themselves have to deal. Of historical science, its elements can only be factually defined, as it in general in regard to method constitutes the antithesis of the purely deductive sciences. It seeks no laws at all from which one can deduce events.[1]

If one now presents the historical method as that of economics, thus this can only mean that it sits near to historical science in relationship of method. It is not entirely without deductive elements, it contains general principles, but these stand in importance far behind the special principles appropriate to the historically given economic picture. Only a small set of concepts can claim universal validity for all economic arrangements, the greatest number changes with them. Thus economics excellently illustrates that incompleteness in regards to principles, since its object is also temporally changeable, while physics, for example, with regard to incompleteness is similar to it, but one in sure sense itself has always the same abiding object, nature. While in this science deduction has a scope independent of time, it is in economics limited to historic periods, within which a system of concurrently valid principles are themselves found. How brief these periods are, only experience can teach. However it extensive may be, so much is the temporal limitedness of the validity of the basic concepts in any case a significant factor that it urges caution in the generalizations of deductions.

There is, however, that yet one factor additionally that significantly limits the applicability of deductive results to reality. Every deduction is necessarily associated with a greater or lesser abstraction. There is no general law that does not at the same time ignore a number of "disturbing" circumstances of reality. The deductive method thus necessarily forgoes capturing full reality. Its results are always only partially valid; mathematics for example only so far as the real things correspond to their idealized conceptions. The warrant of abstraction hangs upon the degree of agreement between its constructs and reality. If the deviations of these from those with sense can be understood as mere intrusions which modify the results only a little, then the absrtraction is scientifically permissible. It provides then an adequate approximation of reality, and we always have to be satisfied with that. For every deduction, however, an exact account must therefore be demanded of the degree of approximation that through it shall obtain. Much as a numerical calculation or statistical tabulation is worthless if one has no assesment of the accuracy of the numbers, so is also any deduction worthless if one does not clearly explain the requirements and thus make the conditions of it applicability recognizable. Exactly on this point so far the most has been missing. One believed that one had done enough

[1] *[580:1]* The so-called historical laws, especially the developmental laws, always belong to special studies; these laws are political, economic, ethical, and so forth.

if one on certain premises constructed a consequent edifice of conclusions in itself or from some analytic formulæ according to the familiar methods had derived new formulæ; but often the mathematically correct conclusions served sufficiently only to disguise incorrectness in premises. The laity's wariness of the mathematical formula is not entirely unjustified. The recording of protracted numeric and formulaic presentation often requires no small effort even for the expert, from which he in advance does not know whether its only reward shall be that the author had made himself and others an x for a u.

It is therefore not sufficient, for the evidence of the applicability of mathematics to economics, to refer to the impressive series of writings that have effected application.[2] One replies with justice to that argument that the possibility to deduce propositions from assumed premises really needed no proof. One could obviously assume that society consisted of nothing but separate houesholds, which interacted with each other only by the means of exchange and at the same time allowed themselves to be guided only by acquisitional interest. On this basis, it is possible to construct a system consistent in itself; but the same obtains from the opposite assumption, that men simply are only coöperatively active and the determination of the product of their activity is not provided by means of the acquisitional interest, but through allocation according to "rational needs". The issue is not about whether such systems are at all possible, but of how nearly they themselves approach reality, present or future, and opinions about this question are as far apart as possible. Some believe either that they have captured reality or, where that's not so, that they've found norms adoption of which can be immediately demanded; others assert that neither the one nor the other construction provides a useful approximation to current or to future reality.

This applies to the deductive method in general. Some deny mathematic deduction in particular even legitimacy of its fundamental prerequisite, the measurability of the basic economic phenomena.[3] Before we hence investigate whether not mathematic deduction cannot be granted a field of application when it appears with more modest claims than commonly occurs, we must answer the question whether the basic economic concepts such as utility, desire, and so forth are measurable quantities.

II.

In accordance with the basic notions that mathematics in recent time has developed about the nature of numbers,[4] we see in the ordinal number and not in

[2] */582:1/* The most complete bibliography on this direction can be found in Irving Fisher, *Mathematical Investigations in the Theory of Value and Prices*, *Transactions of the Connecticut Academy* Vol. IX., in which though, in order to fill the number, two categories of writings are combined, which, as we shall see, must be separated.

[3] */582:3/* Thus, amongst others, [Friedrich Julius] Neumann in volume 48 of this journal [*Zeitschrift für die gesamte Staatswissenschaft*] p. 443n.

[4] */583:1/* See Dedekind, *Was sind und was sollen die Zahlen?* [*What Are Numbers and What Ought They to Be?*] Braunschweig 1888. Kronecker in the *Festschrift* for Eduard

the cardinal number the primary form of the concept of number. In particular measurement is founded upon an ordering of objects in a series by their size or by the size of one of their qualities. This is especially clear with the more primitive, imperfect types of measurement of the day. Thus the determination of the hardness of a mineral is based upon an ordering of minerals in a series by their hardness, by means of the principle that the softer mineral is scratched by the harder one. The grades which upon this way the hardness of the stones are ascribed are simply the ordinal numbers of this series, which only in this way will provide a sure fixedness if it is compared with a reference series, the Mohs' scale. The numbers say thus only that one staone is harder that another; they don't give however the proportion of hardness, so that a stone of hardness 4 would be doubly as hard as one of hardness 2.

The measurement of temperature with the aid of the thermometer stands upon a not much higher level. Even this is only an ordering of the heat sources by means of the length of a quicksilver column that grows with the termperature. The thermometer degrees do not indicate the ratio of the temperatures. One could produce an analogous ordering also directly, by means of heat sensation. One must content oneself then indeed with few disinguishable degrees, perhaps with those that can be easily indicated verbally without aid of numbers. But this ordering still has a shortcoming compared to that by means of the thermometer. It is purely subjective; that is that it hangs on the personal, temporal, and local sensitivity to warmth, while those have objective validity for all who recognize the dependence of the length of the quicksilver column on the temperature. All measurement of psychophysics is a subjective ordering of the sensations according to their intensity, whereby the individual degrees correspond to the just noticeable differences.[5]

Obviously now the elementary economic magnitudes — pleasure and displeasure, utility, desire — are only capable of a merely subjective ordering. All measurement of these is only in the determination of the ordinal number, which arrives at them in a series similar magnitudes. Such series have only subjective significance for those who create them; anyone else will order the same goods more or less differently in accord with his personal system, value higher what those persons placed lower and *vice versa*. Has one on the other hand now a justification to speak of the utility of a good, of the desire for one, and so forth as of specific magnitudes? So long as one holds in mind the special nature of these sizes and only speaks of them in regards to a specific person valuing them, so long as one further treats the possible ordinal numbers which one ascribes to them only as such, and does not ascribe to them the significance of proportional numbers and speak of utilities one twice as large or one half, so long as one finally does not attempt to introduce units of utility and of desire whose existence presupposes that of such relationships, there is no objection to the term "magnitude". One must then also declare temperature and hardness not mag-

Zeller's fiftieth-year anniversary of his doctorate [in 1887]. Also Helmholtz *ibidem*.

[5] *[583:2]* See Wiener, "Die Empfindungseinheit und das Messen der Empfindungsstärke" ["The Unit of Sensation and the Measuring of Sensitivity"] in *Wiedemann's annals*. N.F. Vol. XLVII, p. 659.

nitudes. The effort to ascribe to economic magnitudes the same nature as the extensive magnitudes measurable by units of geometry and mechanics[6] springs from a spurious emulation of natural sciences, from a mistaken notion, as if the objectively measurable magnitudes in all cases are more perfect. This is just as wrong as if one wished to put the sciences into a rank ordering according to the concept of natural-scientific "exactness" and to declare the mathematical-deductive of these as such most perfect. Since mathematic deduction is the ideal of physics, one has made it wrongly into the scientific ideal overall, as if the historic investigation would not forever assert alongside it its right.

What in the natural sciences would be a gross deficiency the subjectivity of the measures, is in economics a basic property of itself, which to desire to eliminate would have no sense at all. Physics seeks to eliminate subjectivity as far as possible; not only does economics tolerate it, but it forms one of its essential foundations. Were the subjective ordering of desire for goods not different for different persons, then an exchange were not at all possible.

Not ever the fact that in economics magnitudes are only appraised — that is only ordered in the imagination and not measured, that is orderable in themselves — may be considered as a deficiency. It may be a source of many practical delusions, as however the apaprent and no the actual utility is the mainspring of economic actions, so economics accepts the appraisals with their errors and leaves to ethics the criticism of these.

We can then summarize our result, that the basic economic concepts represent subjective magnitudes of a specific degree, and hold it as important that this is emphasized. Quantitative definitions of the same, quantitative version of the basic principles can and must be requested in this limited sense. With the requirement of mathematic precision of the concepts, the application of mathematic deduction from them is not necessarily connected. Whether such from mere ordinal numbers themselves can be made and what objective and theoretic worth they have will hence in the example of exchange theory be presented.

III.

Let A have the goods a_1, a_2, a_3, B have the goods b_1, b_2, and both be inclined to exchange of them. A can form 7 combinations of his goods, namely a_1, a_2, a_3, $a_1 + a_2$, $a_2 + a_3$, $a_3 + a_1$, $a_1 + a_2 + a_3$, B only 3 of them, namely b_1, b_2, $b_1 + b_2$. Since any combination from A can be put together with any from B, thus arise 21 possible cases of exchange. We wish to designate each case by our placing the exchanged goods on the left and on the right of a vertical line, so that for example "$a_2 + a_3 \mid b_1$" would represent the exchange of $a_2 + a_3$ for b_1.

A and B should now, each in his own way, order the 21 cases according to

[6] *[584:1]* Fisher (*opere citato*) § 4 tries to do so by defining the relationship between two benefits. He names the usefulness of a good A twice as great as that of B if that of A is equal to that of C and that of B under otherwise identical circumstances is equal to that of $\frac{1}{2}\,C$. He generally takes the benefit of C twice as large as that of $\frac{1}{2}\,C$ and thus contradicts experience as well as his own other assumptions.

the benefit that they themselves expect from each. In each series then each subsequent exchange will be for the chooser of the series more beneficial and hence more desirable than the previous. It is possible thus that the following two series may arise.

A ranks:

Ord. Nº	Exchange	Ord. Nº	Exchange	Ord. Nº	Exchange
1	$a_1 + a_2 + a_3 \mid b_1$	8	$a_1 + a_2 \mid b_2$	15	$a_2 \mid b_2$
2	$a_1 + a_2 \mid b_1$	9	$a_1 + a_2 + a_3 \mid b_1 + b_2$	16	$a_3 + a_1 \mid b_1 + b_2$
3	$a_3 + a_1 \mid b_1$	10	$a_3 + a_1 \mid b_2$	17	$a_2 + a_3 \mid b_1 + b_2$
4	$a_2 + a_3 \mid b_1$	11	$a_3 \mid b_1$	18	$a_3 \mid b_2$
5	$a_1 + a_2 + a_3 \mid b_2$	12	$a_2 + a_3 \mid b_2$	19	$a_1 \mid b_1 + b_2$
6	$a_1 \mid b_1$	13	$a_1 + a_2 \mid b_1 + b_2$	20	$a_2 \mid b_1 + b_2$
7	$a_2 \mid b_1$	14	$a_1 \mid b_2$	21	$a_3 \mid b_1 + b_2$

B ranks:

Ord. Nº	Exchange	Ord. Nº	Exchange	Ord. Nº	Exchange
1	$b_1 + b_2 \mid a_3$	8	$b_1 + b_2 \mid a_3 + a_1$	15	$b_2 \mid a_1 + a_2$
2	$b_2 \mid a_3$	9	$b_2 \mid a_3 + a_1$	16	$b_1 \mid a_3 + a_1$
3	$b_1 + b_2 \mid a_1$	10	$b_1 + b_2 \mid a_2 + a_3$	17	$b_1 \mid a_2 + a_3$
4	$b_2 \mid a_1$	11	$b_1 \mid a_1$	18	$b_1 + b_2 \mid a_1 + a_2 + a_3$
5	$b_1 + b_2 \mid a_2$	12	$b_2 \mid a_2 + a_3$	19	$b_2 \mid a_1 + a_2 + a_3$
6	$b_2 \mid a_2$	13	$b_1 \mid a_2$	20	$b_1 \mid a_1 + a_2$
7	$b_1 \mid a_3$	14	$b_1 + b_2 \mid a_1 + a_2$	21	$b_1 \mid a_1 + a_2 + a_3$

In order that such an ordering of case of exchange is possible, one need not accept that the individual goods are independent in reference to their utility from each other, so that two goods that individually have a low utility also together provide only a low benefit, which obviously does not fit reality in general.[7] Were all goods independent of one another, then were the ordering of all cases of exchange determinate if one only knew the benefit of possessing the individual good.

We have indicated the degrees of benefit or of desire all with positive numbers. It will not thereby be said that all cases of exchange have a positive desire; rather in general an indifference point (null point) will be present and all lower degrees thus negative benefits, that is denote disadvantages. In the next investigation, however, we need not consider this point yet.

The problem of exchange consists now in finding-out those of the 21 possible cases that will be effected, if an exchange is effected at all. Can one under all circumstance designate a case that must be chosen, or what conditions must be met so that a definite decision can be made? To decide this, we arrange the benefits for A and B in the following scheme:

[7] *[586:1]* Fisher's *opere citato* maintains this condition at least in the first part of its developments, thereby limiting its validity by a considerable amount.

	a_1	a_2	a_3	$a_1 + a_2$	$a_2 + a_3$	$a_3 + a_1$	$a_1 + a_2 + a_3$
b_1	6 11	7 13	11 87	2 20	4 17	3 16	1 21
	0	0	0		0	0	
b_2	14 4	15 6	18 2	8 15	12 12	10 9	5 19
	0	0	0	0	0	0	
$b_1 + b_2$	19 3	20 5	21 1	13 14	17 10	16 8	9 18
					*	0	

In each column, on the left stands the ordinal number of the benefit for A, on the right that for B; in the fourth column of the second row, for example, which represents the exchange $a_1 + a_2 \,|\, b_2$, 8 is the degree of desire by A, 15 that by B.

Whatever now the particular motives of the individual may be, still in any case the general economic principle applies that everyone prefers a larger benefit over a smaller one. Thus if an exchange offers both A and B a greater benefit than another, then it will under all circumstances be preferred to this. So the exchange is $a_1 + a_2 \,|\, b_1 + b_2$ for both parties is more advantageous than $a_2 + a_3 \,|\, b_2$ and the latter can hence absolutely be dismissed. In this way one can immediately eliminate all cases that for both traders would provide less benefit than another one possible. The thus eliminated cases we have in the scheme designated with "0". There now remain only 9. If one wishes to find them systematically, the one must construct for oneself a new scheme with 21 horizontal and 21 vertical series, thus with 441 cells, and order the above pairs of numbers according to size. Then the 9 selected cases immediately catch the eye as the outermost occupied positions.

Until then, we had no necessary condition that could not be described as generally fulfillable or achievable. Now that it is about the selection from the 9 above remaining exchange cases, we must introduce a requirement that will not generally be fulfilled by exchange. We assume *that none of the traders will give the other a benefit without remuneration, let alone at their own expense.* This stipulation characterizes exchange in the strictest sense; we can understand it as a definition of actual exchange. So it does not thereby claim that a mutual surrender of goods is possible under no other condition or that any other would be uneconomical, nor that every interchange really complies with the condition, but only a distinction has been made between the cases that meet the condition and the remainder. Only the former should be called exchanges; the others may be referred to as divisions or partial donations or otherwise. It increasingly approaches a gift, the smaller the fee on the one side. A gift itself constitutes thus the borderline case for these interchanges. It can be of two types, each by the value that the given good has for the giver. If it is useless and also unusable for any other purchase, then it needs no moral motives, only in another case does the gift mean a sacrifice for the donor.

Mathematical deduction is obliged to set that condition for exchange, without regard for the scope of its validity; because without it, it can go no step further. For partial or total gifts, there are no general laws; at the most one could state limits of rational generosity, which lie where the use of the donated good for the giver begins to become larger than for the recipient. We will below come back to this point. As to the real validity of the exchange condition, it is in most cases, which are isolated and effected far from a large market, not strictly fulfilled. In a large market however it is in a particular form fulfulled and this fact may justify it if we further follow the

consequences of the assumption.

The first conclusion from it will perhaps provoke reservations concering its validity for market exchange, which we only late can remove; for isolated exchange however one will harmlessly allow its justification as soon as one has accepted the requirement. It is: *The benefits must on both sides be the same*, because only then will neither of them gain a benefit without charge. Each difference in the benefits creates for the one a profit that no equivalent on the other side matches, and if there are amongst the exchange goods none for the possessor absolutely superfluous, thus can even the one can only at cost to the other attain a larger benefit, because whatever more the one obtains escapes the other. If the goods are indivisible, then it is in general impossible to make the benefits exactly equal, and hence then an actual exchange is impossible. One must then either altogether abstain from interchange or on one side reconcile oneself to a surrender. Only by infinitely divisible goods is the requirement under all circumstances realizable.

But its real fulfillment hangs even then upon one further condition. We must, so that about equality and inequality of benefits can be judged at all, make the assumption *that the traders know not only their own benefit but also that of their counterpart and can compare it with theirs.* You must thus be able from the two series on subjective benefits of A and B to produce one single series or rather a double series in which the same degrees oppose each other; then it is good to note that the same ordinal numbers in both series can denote very different degrees. If one wishes, one can imagine the construction of the double series also entrusted to a third impartial person whom both contracting persons know, a broker of trade commissioner, as actually happens in certain cases. The following double series may thus be determined:

A	B	A	B	A	B	A	B
1	—	7	4	14	—	—	14
2	—	8	5	—	9	—	15
3	—	9	—	15	—	—	16
4	—	10	—	16	—	20	17
5	1	11	—	17	10	21	18
—	2	12	6	18	11	—	19
6	—	—	7	19	12	—	20
—	3	13	8	—	13	—	21

according to which for example the 5th degree of B equals the 8th of A, while the 6th of A lies between the 2nd and 3rd of B.

From those 9 number pairs in the previous scheme still available for selection hereafter 17 and 10 indicate the same degree and this case of exchange must thus by the adopted condition come to realization. One sees that, to find it, the listing of a small part of the double series would have sufficed, which series we only for sake of systematic completeness have shown completely.

If there more than two traders present, then only the number itself of exchange cases changes, the principles and the method of the selection can remain quite the same. If there are, for example, 3 traders with each only two goods available, then each of them can distribute his goods in nine ways amongst the three, in which naturally always one must go empty-handed. Since now the goods of each one can themselves be combined with of every one else, then already $9 \times 9 \times 9 = 729$ possibilities of exchange arise in this simple case. From this those must then be eliminated as above which still allow a coincident increase of benefits for all three and from the remainder finally would be the case to select, in which all three benefits are equal, if this happens at all.

It need not be remarked that a trade amongst three or more than three is itself never executed in the described way; but it would be possible that, if also another method of determining the exchanged goods were applied, the result would at least be close to the same. The mathematical exchange theorists have confidence in their reckoning that this, however far the method of price determination may be removed from that customary in the market, delivers the same or rather a more correct result, so that one would better completely eliminate the haggling of the market by letting one market planner calculate according to theory the prices "most advantageous for all". We do not yet have this trust of mathematic theory and would therefore attempt even with regard to the preliminary proceedings to connect ourselves nearer to reality.

With this endeavor we find now that as soon as the number of traders amounts to more than two, there are not only one but two essentially different forms of exchange. We will call them "*communal exchange*" and "*competitive trade*". The overlooking of this situation has had for previous mathematic theory of exchange quite peculiar consequences, to which we below will come back.

What we just previously described was a *communal exchange*. The essence of this consists in that the exchange will be contained in one common, single act of all participants. It can be understood as the combining of all goods of the exchange and a division of the sum thus resulting accodring to stipulation of their contribution to the same, or according to the strict principle of exchange of the equality of benefit for all, or according any other principles of convenience, justice, or equity.

Practically, so far as we known, *communal exchange* exists now in one case, namely as a so-called *Güterbereinigung* [*goods adjustment*]. It is an exchange of divided and scattered acreage to others, which form a possibly rounded complex around the economic center. It is not easy to specify briefly the principle according to which doing this in practice is followed. The process, which incidentally varies with local circumstances, is given roughly as follows.[8]

The entire area is divided into elements of a suitable size and these are distinguished by types of soil and classified according to quality. As far as possible, then first each involved party shall be rewarded for the parcels provided by him by suchlike of such similar type, of similar quality, and at similar distance from marketplace but in united location. — The marketplace may be considered as unchangeable. — Doing this thus in sum those would have the greatest benefit who provided the most parcels in the combination, because each one improves the position. If an award of this sort is no longer possible or not expedient, a reward can also so be made with parcels of more favorable location as those previous but of lower quality. By assessment of the various benefits against each other one seeks to effect a preferably perfect equalization, by compensating for an excess benefit of one sort with a shortcoming at another point. Also with regard to the types of soil, parcels will not always be replaced by similar parcels but it can also, if it can happen without disadvantage for the previous owner, for example grassland can be exchanged for farmland or for woodland. For it is well to note that a separate treatment of the individual parcels is wrong, since none of them has a worth independent of the remaining parcels with which it shall finally be combined into a whole. To the technology of agriculture is thus full consideration to be given. Also, particular requirements and desires of individuals are in accordance with possibility to be considered, and finally a residual is left to proper sentiment and

[8] *[591:1]* Brehme, L., *Der Wegweiser zu einer möglichst zweckmässigen etc. Zusammenlegung der Grundstücke.* [*The Guide to a Most Appropriate etc. Consolidation of Properties.*] Weimar, 1854.

discretion of the commissioner.

The principle of this communcal exchange obviously lets itself be no more precisely formulated than how the real process is. Division according to the stipulation of the previous ownership is arguably the narrowest formula that encompasses all cases. We can ascertain negatively that the interchange principle in any cas is not that of strict exchange, under which the subjectively valued benefits of all are the same; rather they are proportional to the value of the original property, so that perhaps the return to all involved enterprises rose by roughly the same percentage. Perhaps someone might assert that it were better if action were according to the strict principle of exchange, since with absolutely equal benefit all of the benefit of the small owners would be relatively larger; but which commissioner will guage the subjective benefits of all and make them equal? However, even if perfect equality of benefits were to be attained, we don't necessarily want to present this as the ideal, even though the in any case one requirement of the result of the exchange were fulfilled, that they should all be satisfied with it. When none has greater advantage than another, certainly none has rightful cause for dissatisfaction. But this requirement is not the only one that is to be fulfilled, and it is not only in this way to be fulfilled. Not only should the private interests of everyone involved have influence, but above all also the communal interest shall be observed, which possibly is also within the inscribed legal limits to considerable extent. In the related literature, some indications about this are found. Whether the bounds established by justice itself are conducive or hindering to the communal interest will not be examined.

While thus to communal exchange the strict exchange principle can hardly come to application, it seems to be maintained in competitive trade as far as possible. It distinguishes itself from communal exchange essentially per the following circumstances. The interchange forms here not one single communal act, but it resolves itself into a complex of business between each pair. There is hence between the individual exchanged goods always a reciprocity of the sort that one good received by A from another, B, is always given as price, while he alternately from A in return has received a particular good. This reciprocal relationship does not exist in communal exchange, in which the combined mixture of goods represents as it were an intermediate for all exchanges. The essential difference however exists simply in the competition, which is impossible in communal exchange.

The characteristic of competition here particularly coming into consideration is this, that in place of one opportunity of exchange of communal exchange it presents several of them for selection. In communal exchange, each has only two eventualities to hold in sight; either the existing condition remains or the interchange comes-about. If however competition is available, then the failure of an single exchange doesn't mean that of any exchange at all. In this way the considerations of the traders will be essentially different from in isolated or communal exchange. The benefit of an exchange will in general appear small, as it is no longer up against the state before the exchange but an alternative exchange, which likewise offers a benefit, albeit inferior. The notion of the benefit of exchange receives thus here a quite new significance, its being relative to the other opportunities for exchange, especially to the most favorable amongst them. The relative benefit of exchange can even become zero, without that therefore preventing the exchange, because the absolute benefit can yet remain very great. It tends even the greater to be, as the relative becomes ever smaller. With a good of daily need, which we can buy with certainty on the market at a set price, the relative benefit is zero, since we can obtain it indeed always otherwise at the same price. We have therefore no awareness even at all of gaining a benefit through purchase, even though

we immediate become aware of it if we imagine ourselves in the position perhaps to have to achieve through our own work that which we so thoughtlessly acquire through purchase. The absolute benefit of a trained traffic is one quite immeasurable, we are however so completely entangled with it that we only think about the relative benefit, only about whether we cannot purchase a product cheaper elsewhere. Of course, the relative benefit alone determines the purchase price. To overlook this is the second major error of the analytic theory of exchange.

In competitive trades, provided that they meet the strict condition of exchange, only the relative benefits of both traders are thus equal for each exchange. For different exchanges they can be very different. The absolute benefit remains completely unknown and without effect. If we will also to competitive trade apply the symbolism introduced above, then we must determine in each range of benefits the point of indifference, not as above as the one point that indicates the disappearance of benefits in general, but as the one that indicates the greatest otherwise reachable benefit. One numbers the degrees of benefit from this point, thus one obtains the degree of relative benefit. For example, the next most beneficial exchange that A can complete instead of the exchange $a_1 + a_2 + a_3 \,|\, b_1 + b_2$ with B is the exchange $a_1 + a_2 \,|\, c_1$ with C and $a_3 \,|\, d_1 + d_2$ with D, thus the degree between both cases measures the relative benefit of the former exchange. The relative zero-point shifts from case to case.

How far the requirements of equality of the relative benefit in competitive trade in general is satisfied, we don't wish to examine. With greatest approximation, it is in any case satisfied in the most important special case of the same, in the market exchange. Of one such thing we speak where the exchange goods are originally determined in wares, that is for the exchange, and offered in large amounts with wide competition and desired goods available. It is added thereto that in market-exchange money functions as a generally medium of exchange and price-measure. Thereby especially will one such simplification of the combinatorics of opportunities for exchange be effected, that these may themselves be assessed and managed practicably.

The result of the application of the general principles of exchange to market-exchange is forseen: It must itself produce the well-known law of supply and demand. We will still derive it systematically; partly to make clear its connection with the previous one, partly to gain a further reference point for criticism of analytic exchange theory.

We consider first the buyer. He has to consider which combinations of goods for his stock of money are possibly purchasable. Each constitutes a case of exchange in the sense set-out above. The benefit of each is a different one. The permit themselves to be ordered by benefit, whereby the difference of the absolute and relative benefit is still not at all under consideration. How the available money can be directed hangs obviously on the prices of the wares. We will denote the arrangement of the prices of all purchasable wares in short as a "tariff". To each tariff belongs a specific number of combinations and under these is always one which is most beneficial. Only this comes under consideration, so that thus the buyer only needs to effect of his stock of money the most beneficial disposition belonging to all possible tariffs. He can even content himself provisionally to effect on the most beneficial disposition corresponding to the presently prevailing tariffs, with the proviso to change it whenever the tariff assumes a different arrangement.

Let us assume now that all buyers have in this way effected their disposition. It were then determined in reference to each type of good which buyers are economically

able to pay for these[9] and how large a sum they have determined for purchase of these. It stands thence also the quantity of wares of the relevant type which there is a call with willingness to pay is fixed. We measure the size of the demand by this quantity of wares and may hence briefly also refer to this as the "demand". As the disposition of the buyer if his money was made and had to be made without consideration of the actually available wares, since to each individual the quantity of wares coming onto the market is unknown, so in general the demand will not agree with the actually available quantity of wares.

The *seller* will, like the buyer, depending on the tariff make a disposition of his stock of goods, by partially bringing it to market, deciding partially to hold back until a more favorable opportunity. We can imagine his freedom of disposition to be increased if he also has some more influence over the production or import of the wares. Amongst the possible dispositions for each tariff, also here one will always be the most beneficial, which alone is to be considered.

Now, in accordance with the standing tariff, the totality of sellers have determined for themselves which ware each of them will put on the market, thus thereby for each type of ware the quantity is set, which by the given price from them will be offered for sale. By the amount will the supply of the relevant amount of the type of ware be measured and it also itself briefly called the "supply".

If the price of a ware rises then in general a smaller number of buyers will be able to pay or at least the same buyers only for a smaller amount of the ware, whereas the sellers in general will bring more wares on the market. The demand decreases; the supply increases. With dropping prices, it holds contrarywise. If we now assume *that the market is large enough for this interaction in each case with certainty to be expected*, so one is always able to find a price-range within which the supply and the demand are equal. If this price is reached for all types of wares, then there is equilibrium in the market. It always to this state tends. Each buyer can then satisfy his entire desire to buy and no seller will hold an unmarketable residual of wares. But, still further, each buyer can at the equilibrium price obtain the wares from all competitors at the same price and each seller can without any difference in price sell dispose of them with any buyer; in other words the relative benefit for each individual purchase is equal to zero. With trade in the market, basically no one makes thus profit; *there are equal values exchanged*. Since all relative benefits are zero, so they are thereby also equal, thus the condition of exchange in strictest form is fulfilled.

One has repeatedly criticized the market law, in particular *[William Thomas]* Thornton[10] has produced a series of examples in which the price itself did not change despite changing of the offered or demanded amounts of wares. However, these examples prove that the law of supply and demand is no general law of exchange, for which it is commonly given. We have expressly stressed its conditions for validity. If through reduction of price the demand does not increase or the supply is not reduced, which with a small number of agents will very often occur, then can also supply and demand not be made equal through variation of the price. The meaning of the terms "supply" and "demand" as certain quantities of wares at specific prices is here good to heed. If one undertands thereunder any indeterminate intensities,[11] then it is impossible to establish any law at all.

[9] *[597:1]* P. 22 of this annual volume [*Zeitschrift für die gesamte Staatswissenschaft* v 49].

[10] *[599:1] Die Arbeit* [*On Labor*], German by [Heinrich] Schramm, p 51*ff*.

[11] *[599:2]* [Friedrich Julius] Neumann in [Gustav von] Schönb[erg's] *Handb[uch der politischen Ökonomie]*. 2nd ed., p. 287*ff*.

One further law only valid for market activity is that of the *proportionality between the quantity of wares and price*. It has its foundation in the divisibility of wares and in the possibility to buy and to sell either in large or small according to which one or the other showed itself more beneficial. Were now the price of a larger quantity relatively smaller than the retail price, then would the endeavours of the buyers go thereby to buy the largest possible quantities, while it, apart from other circumstances, would lie in the interest of the sellers to sell in the smallest possible amounts. The tendency of the buyer, however, acts against that of the seller. If they maintained perfect equilibrium, then would proportionalist of amount of wares and price obtain. Now, however, the endeavors of the buyers to increase the purchase quantity have in their ability to pay for this a natural limit, which the sellers on the other hand is prevented from effecting an arbitrary reduction of the quantity due to the rising costs of retail business. All these factors act, according to experience, together then so that there is a moderate decrease of the prices as increasing quantities of wares come to stand, and within certain bounds perfect proportionality obtains. A decrease of the prices with decreasing quantity is complete out of the question, because neither buyer nor seller has an interest in that.

It was necessary also to analyze these simple relationships, because the mathematic exchange theory, to whose consideration we hereafter turn, of the law of proportionality also makes a very uncritical use.

IV.

The task of the natural sciences is the cognizance of an area of reality completely independent of the will and action of mankind. You can in the solving these use any method, which at all, even if on still more roundabout ways, leads to the goal. The political sciences wish to comprehend human desire and will. Knowledge and reality themselves lie here much nearer than there. The comprehension of economic processes is an analogue of the processes themselves; knowingly we live through it, as it were. The method of knowledge, not just the result of it, closes itself upon reality and, as ever nearer it does it, all the more perfect is it. Each theoretic step should possibly correspond to an analogous process of economic action. The train of thought should itself in all its parts be able to be applied immediately to reality.

We believe in the previous section to have reached but also not to have exceed the outmost limit. From the symbolism we could at most have abstained yet without injury; that is a matter of taste. Our course was one purely synthetic. Any analysis that strikes paths on which reality cannot accompany it exceeds the limits necessary to be set. Only the beginning and end of the analytic contemplation stands, if it was correctly effected, on firm ground; the connecting bridge lifts itself above it. We hold analytic calculation in economics permissible only where they also in reality occur. Where the banker or the financial politician reaches to analysis to calculate profit or yield, there may also the theorician follow them; what is beyond it, that of from evil. This is proven best by analytic exchange theory itself.[12]

It seeks only to be a theory of market exchanges, which certainly is not always said explicitly. In some representations[13] indeed seemingly the exchange of two in iso-

[12] *[600:1]* The theories based themselves on graphic representation are not essentially different from it, but are founded on the same principles.

[13] *[601:1]* For example, in Jevons, Launhardt and others. Walras emphasizes that he only ever talks about competitive exchanging.

lation is presumed; but we find everywhere applied the law of proportionality between quantity of wares and price, which yet holds only for the large market, and we must hence not understand the elementary case chosen as the starting point as an isolated exchange, but as an individual exchange in the market. Before we now further investigate whether in what remains the treatment is correct, first should be shown that the restriction to market exchange for analytic representation was not at all necessary, since one does not need the law of proportionality. We can concerning the prerequisites certainly commit ourselves to our synthetic representation, which actually arose from isolated exchange. Only on one point are we forced to a deviation. We must presume goods, if not differentiable, then nonetheless reducible into small, *homogeneous* parts, if the analytic functions and their derivatives even only approximately should represent the circumstances of reality. This requirement is a considerable restriction of the validity of the formulæ. Indivisible goods must be left completely aside.

Now A has a good fulfilling this condition, of which we will indicate a variable quantity with "x". Also B has only a divisible good, of which a variable amount will be indicated y with "y". x and y may be exchanged for each other. The benefit of exchange for A will be all the greater as y is greater and as x is smaller; that for B contrariwise, as the smaller y and as the greater x is. We will further assume that these benefits themselves can be represented as analytic functions of x and y, which even if the benefit should be expressed only in ordinal numbers is not impossible, and will indicate the benefit for A with "$V_a\,(-x, y)$", that for B with "$V_b\,(x, -y)$", where then "V" in both cases denotes a function of these variables, increasing with x and y.

We have to seek now the values of x and y that correspond to the cases of exchange that mark the limit-points of the simultaneous growth of both benefits. If we imagine V_a and V_b graphically as ordinates of two areas lying on the same plane, such allows this task to be thus expressed: We seek in both areas those corresponding to each other, that is points lying over the same points of the common base plane, of which a climb on one surface without a descent on the other is not possible.[14] These would correspond to the 9 points that in the example treated above according to the first selection were left remaining. If the points on the areas are of a normal sort, then there is at each point a horizontal tangent that separates the paths of ascent from those of descent. If we take any two corresponding points on the areas, the their horizontal tangents will in general cross each other. Amongst such points four cases can occur, each according to the direction in which one proceeds: V_a and V_b can at the same time grow, both can at the same time decrease, V_a can grow and V_b decrease, and the opposite can occur. Only if the two horizontal tangents are to each other parallel do either the first two cases apply with the exclusion of the last two, or contrariwise these with the exclusion of the first. Under the second eventuality is thus a simultaneous increase impossible, and thus parallelism of the horizontal tangents is in any case a necessary condition, even if not a sufficient one, for the sought pair of points. If we had found those in which the two horizontal tangents were parallel, then it would be a matter of further investigation whether the first or the second eventuality were present.

The points will be analytically found, however, as one differentiates V_a and V_b with respect to x und y, sets the differentials equal to zero, and eliminates dx und dy from them, because on the one hand the differentials of the area coördinates must in the desired direction be zero, and on the other hand dx und dy have the same value in

[14] *[602:1]* See my article "Eine Erweiterung des Maximumbegriffs" in the current annual volume of *Zeitschrift für Math[emik] und Phys[ik]* [v 38 (1893) pp 315-17].

both. One finds thus from

$$-\frac{\partial V_a}{\partial x}\cdot dx+\frac{\partial V_a}{\partial y}\cdot dy=0\ ,\ \frac{\partial V_b}{\partial x}\cdot dx-\frac{\partial V_b}{\partial y}\cdot dy=0$$

$$\frac{\partial V_a}{\partial x}\cdot\frac{\partial V_b}{\partial y}=\frac{\partial V_b}{\partial x}\cdot\frac{\partial V_a}{\partial y}$$

as the equation of a curve in the base plane, which possibly describes the desired points. Whether it does it hangs upon the inclination of the two tangent planes of the points to the base plane. If both form acute angles with it in the same sense, then there are in any case no values of x and y that meet the first condition of exchange. If on the other hand one forms an acute angle, the other an obtuse one, then the points are found, from which a simultaneouse increase of V_a and V_b is impossible.

This mathematic excursion was necessary because one very blithely passed over this point regularly, as one spoke always without further ado about a maximum where a differential was set to zero. The points of the curve found designate no maxima in the usual sense, not even if they completely fulfill the requirements. We have it rather to deal with a borderline of a quite peculiar type, that with the maximum has only this in common, that it represents exactly a border of progress if this is linked to certain conditions. If both traders go so far that a further increase in the benefits of one cannot take place without penalty to the other, then neither one nor the other has reached the maximum of his benefits, but it hinders rather each other from reaching the real maxima while the interests on both sides oppose each other. But we will, before we go further into these popular devices of maximization, finish first the problem of isolated exchange.

We have first found an equation between x and y. To determine these values, a second is necessary. This gives us the exchange condition of the equality of benefits,

$$V_a=V_b\ ,$$

which, since x and y were assumed to be divisible, always is fulfilled.

The introduction of several types of wares on both sides makes the problem not much more complex. If A and B have two types of wares, then one can represent their benefits with

$$V_a\left(-x_1,-x_2,y_1,y_2\right)\text{ and }V_b\left(x_1,x_2,-y_1,-y_2\right)\ .$$

The resulting equations are sufficient to determination of the unknowns, if one includes $V_a=V_b$, and thereupon alone some theorists seemed to arrive. The same applies if the number of traders increases. Mathematic difficulties are not present.

But what does it mean if we assume for everyone of those involved in the exchange a single function of the goods V_a, V_b, V_c, and so forth, which represents the benefit of each revelant person when trading? It means that we only consider the absolute benefit and that we do not take note that the benefit of exchange is transformed by competition into a relative one. One must, if one will consider this, not only assume for each peson a peculiar benefit function, but construct this differently for each individual exchange, and make all these relative benefits in quite specific way dependent on each other. This problem, not entirely simple to formulate mathematically, has, so far as we know the literature, been posed by no one, and there is a noteworthy result, that the entire previous theory *has treated of the theory not of competitive trade but, with naïve thoughtlessness, of communal exchange*, which the accompanying remarks about competition don't change in the least. Consequently, this example becomes an

excellent illustration to our remark made above that the analysis unavoidably leaves the guideline of reality and loses thereby permanent control over its steps. So it happens readily that the reckoning controls the reckoner, and that the fair words that accompany it don't agree with the content of the formulæ.

The error is now more completed in this way, that the conditions of communal exchange are not purely sustained but that, beside the absolute benefits, the law borrowed from market trade of the proportionality of amount of wares and price participates, so that in reality *the formulæ combine in a motley jumble of market trade and communal exchange*. That this mathematic deduction is removed very far from reality needs after that surely not particularly to be proven. The calculated numeric values are not in general checkable by experience, and therein lies it that in this way the error cannot be revealed; and, since also the derived general propositions were general enough to escape from concrete criticism; thus the proof of a contradiction in the premises is perhaps the only way to expose the error.

Of more general interest however as such a mathematic error is another illusion that ties into the whole method. The entire calculation aims thereupon to find the so-called *maxima*. Trade with free competition should, according to the presentation of many exponents of the mathematic school, be the form of traffic that for all involved ensures the maximum benefit. This one believes exactly through mathematic deduction to have been demonstrated. One differentiates, sets the differential to zero, and calculates; thus one obtains a maximum; that is the simple conclusion always repeated. We will disregard the mathematic deficiencies of this conclusion, upon which we above touched; we will assume that the resultant values are at least relative maxima in the sense of mathematics; we will leave disregarded, further, the mixing of the two forms of exchange; is it then really in the given way proven that the benefit of eveyone from the underlying form of traffic is geater than from any other? Is this after all in this way demonstrable? One compares indeed really not various forms with each other; one overlooks, as we saw, even the existence of other forms.[15] One examines only how on the basis of specific forms of traffic the benefits to all are best, or actually only how they ultimately can be combined. Whether for example competitive trade itself is more beneficial than communal exchange is really not examined, and cannot by this method really be examined. That must be left to experience. The situation is mostly that ultimately no choice exists between the forms, but one by the circumstances is categorically caught. Otherwise one can also put of traffic in consideration a third form that should form the end of our mathematical considerations. It is indeed in itself not necessary that the goods received by anyone stand in any relation to those that he contributed. All could indeed put their goods together, amongst them again so to distribute that the *sum* of the benefits to all becomes a mathematical maximum. Analytic handling of this problem doesn't stand in the way. The differentiation of $V_a + V_b + V_c + \ldots$ gives

$$\left(\frac{\partial V_a}{\partial x} + \frac{\partial V_b}{\partial x} + \frac{\partial V_c}{\partial x} + \cdots\right) \cdot dx + \left(\frac{\partial V_a}{\partial y} + \frac{\partial V_b}{\partial y} + \frac{\partial V_c}{\partial y} + \cdots\right) \cdot dy + \cdots = 0$$

and it becomes evident that dx, dy, and so forth from each other are independent; this equation only holds when the coëfficients are individually equal to zero. There

[15] *[605:1]* A counter-proof, as Launhardt attempted in *Math[ematische] Begründung der Volksw[irtschaftslehre]* [*Mathematical Principles of Economics* (1885)] §10 is hence in advance inappropriate.

are as many equations as variables available, thus a system from which all quantities of goods can be calculated.

So one can thus with the same evidence prove in a mathematic way that division according to reasonable needs provides a maximum of benefit for the whole, with which otherwise is proven that exchange by free competition brings each participant the greatest possible benefit. The worth of all of this maximization proof cannot be made clearer.

V.

Our investigations into the measurable quantities in economics had a significant gap if we did not yet bring the concept into the field of our consideration that can stand as the economic concept of measure in preëminent sense, the concept of value. At the forefront of discussion of this much used concept should be placed that there are units of value that one thus can investigate how many time as large a value is as another and can replace goods of the same value with each other, that thus the value has a real measure expressible in a cardinal number. If this formal difference between *value* and the concepts used in price theory of desire and benefit is established, then already are some confusions and errors prevented. But the formal characteristic can also lead us to the essence of the matter.

The substitutability of equal values by each other is, like the transferability of the same length to each other the requirement of length measurement, the requirement of measuring value and in it lies at the same time also the essence of the concept of value.[16] How one can just define the length of a line as one indicates what is the same length — namely lines that one can make coincident point by point or successively congruent with each other — so can one also just the value define as one defines equal values. Equal value for an economy have such goods that without changing the net income for the economy can be substituted for each other. So that in fact everything is expressed that characterizes value. It is stated in reference to which attribute the same values are the same — because being the same is a very ambiguous concept — that they are *the same in regard to their importance for the profit of the economy*, in regard to their capacity, to give this a specific height, to increase it and to decrease it.

To undeveloped traffic, the applicability of the concept of value is very limited. Goods are only substitutable with each other in larger groups and there is rarely for that real reason. Only when the traffic gets livelier, ever more individual goods are brought out, which always and everywhere without influencing the yields of the economies can be exchanged with each other. This forms the exchange value, the market value. In accordance with the theory of market exchange, we find also in this way that on the large market — and only on this — equal values are exchanged without benefit or cost to both traders. The exchange in itself does not prove the equality of the values of the exchanged goods, because through a favorable opportune exchange as also by communal exchange the purpose and consequence of the exchange is indeed an increase of economic income. Ultimately, the general medium of exchange and price measure of the market becomes also the value measure of those goods that do not or do not regularly participate in circulation, as these can by trafficked goods of equal yield be replaced.

[16] *[606:1]* [Johann Karl] Robertus[-Jagetzow] (*Z[ur] Erk[enntnis] uns[erer] staatsw[irtschaftlichen] Z[ustände]* [1842]) already bought up this thought.

The economic measure of value loses also through exchange its subjectivity. Equivalent goods bring indeed in different economies by no means equal profits, but two goods of the same exchange value are necessarily equivalent for all economies, that is that they can in any economy be substitute for each other.

This theory of value, which we also already have presented in previous papers,[17] does not break with tradition in a harsh way like the theory of marginal utility, rather fits unforcedly supplemental to the prevailing way of thought. Only the so-called *classic* theory of value, according to which value is a *product* of economic or even technical activity, stand likewise as unsympathetic as it opposed to the marginal utility theory. This owes its dissemination for the most part to the mathematic school. The concept of the last differential was as if made for analytical treatment, and so it has over it risen a stately edifice of formulæ. The concepts thereby expressed required a new terminology, which then also without regard to origin was established. New concepts require new names, but one may not apply old and, in a certain sense, common names to new concepts. We will not here examine the theoretical fruitfulness of the concept of marginal utility. It may be of the greatest applicability. In any case, however, it is not identical to the concept of value, nor value to marginal utility proportional. The view that the use of the last part determines the value of the whole good has main support in the mathematical theory of exchange. According to the formulæ given above, the exchange takes place only on condition that

$$-\frac{\partial V_a}{\partial x} \cdot dx + \frac{\partial V_a}{\partial y} \cdot dy = 0 \text{ or } \frac{\partial V_a}{\partial x} \cdot dx = \frac{\partial V_a}{\partial y} \cdot dy$$

and similar equations apply to the benefits of all participants. The two sides of the equation represent the marginal utility of x and y, thus the exchange only takes place if the marginal utilities are equal and thereby one has concluded that the marginal utility determines the exchange value and therewith the value in general, in that goods of equal marginal utility also have equal value. However now, goods exchanged in general are not necessarily equivalent, but this only applies to the market. Here at least exusts thus a relationship between marginal utility and exchange value? Even this hope fails, because V_a means indeed only the relative benefit of trader A and this is, on the market, as we saw always zero. Hence, the above equation loses for the market any sense at all, because if $V_a = 0$ natually also any differential of V_a is zero and the differential equation becomes completely meaningless.

A further circumstance that may have contributed much to the dissemination of the theory of marginal utility is that it undertook to eliminate the apparent conflict between value and benefit. It acknowledged that the benefit of some goods, such as water, were very great, that however because of the abundance the utility of the last part will be very small, since it only satisfies a very small and not urgent need. We believe, that the conflict can in a simpler and more natural way be resolved, if one relates the benefit always to a concrete quantity of the good.[18] Then the use of a part of a good available in abundance is itself very small and there is no need at all for the often inapplicable differential concept. Incidentally, the remark is sufficient that that the marginal benefit decreases continually with increasing amount of a good, while however surely the value of the total amount of the good does not decrease with the amount, even if it doesn't increase proportionally to the amount, irresistably to demonstrate the untenability of the equating of value and maginal utility.

[17] *[607:1]* Volume 48 and especially volume 49 p. 262 of this journal [*Zeitschrift für die gesamte Staatswissenschaft*].

[18] *[609:1]* See volume 48, page 202.

We are at an end and want to summarize the result of our research that a use of measure and number in applications is absolutely to be desired; in principles for their use however the following limitations are to be made: One part of the elementary quantities have no measure at all expressible in units, but there are only differences in degree that can be expressed in ordinal numbers. This does not prevent them from being interpreted as quantities and quantitatively defined. Actual mathematical deduction is to be applied to them only in closest connect to reality, hence according to the synthetic method, and then only under most careful consideration of the conditions of validity. The analytic method, together with the graphic method used with it, is not only dispensible but also misleading and has until now, at least where price theory is concerned, only led astray. Value is the actual economic measure of goods. The marginal utility theory is to be discarded as a price theory.

Translator's Notes

This document is merely a first pass translation; it surely contains many errors.

It is my intention that the final version will be in proper English, but otherwise preserving as much of Voigt's expression as reasonable. I do not wish to sacrifice fidelity to make the translation more readable.

The content of each footnote is prefaced with a bracketed indication of its original page number and its number on that page. (In the original, numbering of footnotes began anew on each page.)

I have added borders around the nine most significant pairs in the final table of section III.

I have modified the mathematical notation in formulæ to indicate all multiplication explicitly.

Queries and comments may be directed to me at `Mc_Kiernan@oeconomist.com`.